

Contents

Abstract	iv
Dedication	v
Acknowledgements	vi
Publisher Notes	vii
Preface	1
Introduction	3
1 Preliminaries	7
1.1 Maxima and Minima	7
1.2 Calculus of Variations	9
1.2.1 Euler-Lagrange Equation	11
1.2.2 Beltrami Identity	11
1.2.3 Lagrange Multiplier	11
1.3 Wrap Up	12
1.4 Bibliography	12
2 The Cycloid Curve	13
2.1 History	13
2.2 Parametric Equation for a Cycloid	14
2.3 The Cycloid In Differential Equation Form	15
2.4 Isochronism	16
2.5 Bibliography	18
3 The Catenary Curve: A Suspended Cable	19
3.1 Problem: Derive the Suspended Cable Curve	19
3.2 History	19
3.3 Solution	20
3.4 Bibliography	24
4 The Brachistochrone Curve: Fastest Descent	25

Contents

4.1	Problem: Derive the Curve of Fastest Descent	25
4.2	History	25
4.3	Solutions	26
4.3.1	Direct Gravity Argument	27
4.3.2	Johann Bernoulli's Solution	29
4.4	Bibliography	31
5	The Tautochrone Curve: Equal-Time Descent	32
5.1	Problem: Isochronous Curve Derivation	32
5.2	History	32
5.3	Solution	33
5.3.1	Gravity Solution	33
5.4	Bibliography	35
6	The Geodesic Problem: Minimizing Distance	36
6.1	What is a Geodesic?	36
6.2	On The Plane	37
6.3	On The Sphere	38
6.3.1	The Haversine Formula	42
6.4	Bibliography	42
7	The Isoperimetric Problem: Maximizing Area	43
7.1	History	43
7.2	Solution	44
7.3	Bibliography	47
8	A Proof That $\Pi = 4$?	48
8.1	A Paradox	48
8.2	An Explanation	50
8.3	Bibliography	51
9	Commentary	52
9.1	What Makes A Cycloid So Special	52
9.2	Why is The Catenary Curve Not A Cycloid?	53
9.3	Bibliography	55

Preface

It is both mysterious and true that the the laws that govern the universe are deeply mathematical; at least so far as we understand them. It is also true that even today literally no one understands precisely why that is the case. Can a universe exist that is not mathematical? Can such a universe evolve life intelligent enough to even ask the question? Again, no one knows.

But what we do know is that The Calculus plays an enormous role in our ability to understand, predict, and navigate the universe around us. It has done so for centuries. The breadth and influence of calculus in modern technology is vast and largely hidden from view. Most of its historic antecedents are lost to those who do not study its evolutionary history. It is deeply embedded in modern technology and hidden (or obscured) from view by user-friendly user interfaces.

In this monograph we recapitulate some of that history by gathering into one volume a few of the interesting historical problems that motivated the great scientific minds of the 17th and 18th century. In particular, the presented problems involved determining the equation of a curve that accurately modeled a particular well defined physical phenomenon. These investigations stimulated the minds of the greatest polymaths of their era and laid bare their intuitive genius. It should be helpful and inspirational for an interested student of the calculus to appreciate the insight brought to bear on early mathematical physics applications at the dawn of modern technology.

To be sure, this short monograph is but a mote¹ in the large eye that is the remarkable history of The Calculus. Its intended audience are those versed in elementary differential and integral calculus and have either a recreational or early-academic interest in mathematics. It presents and analyses a selection of historically important physical and mathematical challenges - both physical and imaginative - whose derivation and solution challenged the leading natural scientists of their day and affected an important step forward in their time with respect to applications of calculus to the real world.

As for the expected mathematics background: a familiarity with elementary differential and

¹A tiny piece of substance. An oblique reference to *The Mote in God's Eye* (1974), Pournelle & Niven.

Variations On Curves

integral calculus should suffice in order for the reader to follow the conceptual development contained herein. A Preliminaries chapter provides an overview of the important concepts that are fundamental to understanding the motivation for the solutions presented in the main text.

Also, it should be mentioned that in many of the derivations and constructions to follow, we “reason” with differentials as was done by the 18th and early 19th century savants whose works we are investigating. That is, we play fast and loose with differential algebra. Hopefully this less than fully rigorous treatment will not offend. What it allows is a fast-paced (and hopefully stimulating) investigation of each problem without an excess of formal apparatus.

To be clear, this monograph is simply a compilation, evocation and exposition of a collection of historically important and intrinsically interesting technical challenges and how the computational power of calculus was brought to bear on their solution and resolution. The author consulted numerous on-line and physical text resources to put together this compilation. Every attempt was made to reference relevant external sources within and appropriate to each chapter. In this fashion each individual chapter stands on its own and may be read independently from other chapters.

It is hoped that this presentation will also be appreciated as a celebration of the marriage between physics and the language it speaks - mathematics.

Lastly, the author very much hopes this exposition will arouse and excite young wannabe mathematicians; the audience to which this presentation is dedicated.